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BASIS PROPERTIES OF p–EXPONENTIAL FUNCTION OF LINDQVIST AND PEETRE TYPE

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Abstract. We show that a p-exponential function defined by the p-trigonometric functions of Lindqvist and Peetre form a basis in the Lebesgue space $L^r\big((-1,1)^n\big)$ for any $r \in (1,\infty)$, provided $n \leqslant 3$ and $p > p_n \geqslant 1$.

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