

SOME INEQUALITIES FOR MEANS DEFINED ON THE LORENTZ CONE

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Abstract. In this paper, we define various means associated with Lorentz cones (also known as second-order cones), which are new concepts and natural extensions of traditional arithmetic mean, harmonic mean, and geometric mean, logarithmic mean. Based on these means defined on the Lorentz cone, some inequalities and trace inequalities are established.

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