OPTIMAL WEAK PARALLELOGRAM CONSTANTS FOR L^p SPACES

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Abstract. Inspired by Clarkson's inequalities for L^p and continuing work from [5], this paper computes the optimal constant C in the weak parallelogram laws

$$\|f+g\|^r + C\|f-g\|^r \leqslant 2^{r-1}\left(\|f\|^r + \|g\|^r\right),$$

$$\|f+g\|^r + C\|f-g\|^r \geqslant 2^{r-1}\left(\|f\|^r + \|g\|^r\right)$$

for the L^p spaces, $1 < p < \infty$.

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REFERENCES

[1] W. L. BYNUM, *Weak parallelogram laws for Banach spaces*, Canad. Math. Bull. **19** (3): 269–275, 1976.
[2] W. L. BYNUM AND J. H. DREW, *A weak parallelogram law for l_p*, Amer. Math. Monthly **79**: 1012–1015, 1972.
[3] N. L. CAROTHERS, *A Short Course on Banach Space Theory*, volume 64 of London Mathematical Society Student Texts, Cambridge University Press, Cambridge, 2005.
[4] R. CHENG, A. G. MIAMEE, AND M. POURAHMADI, *On the geometry of $L^p(\mu)$ with applications to infinite variance processes*, J. Aust. Math. Soc. **74** (1): 35–42, 2003.
[5] R. CHENG AND W. T. ROSS, *Weak parallelogram laws on Banach spaces and applications to prediction*, Period. Math. Hungar. **71** (1): 45–58, 2015.
[6] RAYMOND CHENG AND CHARLES B. HARRIS, *Duality of the weak parallelogram laws on Banach spaces*, J. Math. Anal. Appl. **404** (1): 64–70, 2013.
[7] RAYMOND CHENG, JAVAD MASHREGHI, AND WILLIAM T. ROSS, *Birkhoff-James Orthogonality and the Zeros of an Analytic Function*, Comput. Methods Funct. Theory **17** (3): 499–523, 2017.
[8] RAYMOND CHENG AND WILLIAM T. ROSS, *An inner-outer factorization in ℓ^p with applications to ARMA processes*, J. Math. Anal. Appl. **437** (1): 396–418, 2016.
[9] J. A. CLARKSON, *The von Neumann-Jordan constant for the Lebesgue spaces*, Ann. Math. **38** (1): 114–115, 1937.
[10] MAHLON M. DAY, *Normed linear spaces*, Academic Press, Inc., Publishers, New York, Springer Verlag, Berlin-Göttingen-Heidelberg, 1962.
[11] JI GAO AND KA-SING LAU, *On the geometry of spheres in normed linear spaces*, J. Austral. Math. Soc. Ser. A, **48** (1): 101–112, 1990.
[12] OLOF HANNER, *On the uniform convexity of L^p and l^p*, Ark. Mat. **3**: 239–244, 1956.
[13] R. C. JAMES, *Orthogonality and linear functionals in normed linear spaces*, Trans. Amer. Math. Soc. **61**: 265–292, 1947.
[14] HONGWEI JIAO AND BIJUN PANG, *Pythagorean parameters and normal structure in Banach spaces*, JIPAM. J. Inequal. Pure Appl. Math. **9** (1): Article 21, 4, 2008.
[15] P. JORDAN AND J. VON NEUMANN, *On inner products in linear, metric spaces*, Ann. of Math. (2), **36** (3): 719–723, 1935.
[16] ATTAPOL KAEWKHAO, *The James constant, the Jordan-von Neumann constant, weak orthogonality, and fixed points for multivalued mappings*, J. Math. Anal. Appl. **333** (2): 950–958, 2007.

[17] YOUNG CHEL KWUN, QAISAR MEHMOOD, WAQAS NAZEER, ABSAR UL HAQ, AND SHIN MIN KANG, *Relations between generalized von Neumann-Jordan and James constants for quasi-Banach spaces*, J. Inequal. Appl., pages Paper No. 171, 10, 2016.

[18] JUAN JORGE SCHÄFFER, *Geometry of spheres in normed spaces*, Marcel Dekker, Inc., New York-Basel, 1976, Lecture Notes in Pure and Applied Mathematics, No. 20.

[19] FENGHUI WANG, *On the James and von Neumann-Jordan constants in Banach spaces*, Proc. Amer. Math. Soc. **138** (2): 695–701, 2010.

[20] CHANGSEN YANG, *A note on Jordan-von Neumann constant and James constant*, J. Math. Anal. Appl. **357**: 98–102, 2009.

[21] CHANGSEN YANG AND FENGHUI WANG, *On a new geometric constant related to the von Neuman-nJordan constant*, J. Math. Anal. Appl. **324**: 555–565, 2006.