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LIMITING CASE HARDY INEQUALITIES ON THE SPHERE

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Abstract. We give sharp limiting case Hardy inequalities on the sphere $\mathbb{S}^2$ and show that their optimal constants are unattainable by any $f \in H^1\left(\mathbb{S}^2\right) \setminus \{0\}$. The singularity of the problem is related to the geodesic distance from a point on the sphere.

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