

## TURÁN TYPE INEQUALITIES FOR $q$ -MITTAG-LEFFLER AND $q$ -WRIGHT FUNCTIONS

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*Abstract.* Our aim in this paper is to derive several Turán type inequalities for the  $q$ -Mittag Leffler and  $q$ -Wright functions. Moreover, we prove the monotonicity of ratios for sections of series of  $q$ -Mittag Leffler and  $q$ -Wright functions, the results is also closely connected with Turán type inequalities. In order to obtain some of the main results we apply the methods developed in the case of classical Mittag–Leffler and Wright functions. At the end of the paper we pose two open problems, which may be of interest for further research.

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