NEW INEQUALITIES FOR OPERATOR CONCAVE FUNCTIONS INVOLVING POSITIVE LINEAR MAPS

SHIVA SHEYBANI, MOHSEN ERFANIAN OMIDVAR AND HAMID REZA MORADI

Abstract. The purpose of this paper is to present some general inequalities for operator concave functions which include some known inequalities as a particular case. Among other things, we prove that if $A \in \mathscr{B}(\mathscr{H})$ is a positive operator such that $mI \leqslant A \leqslant MI$ for some scalars $0 < m < M$ and Φ is a normalized positive linear map on $\mathscr{B}(\mathscr{H})$, then

$$\left(\frac{M+m}{2\sqrt{Mm}} \right)^r \geqslant \left(\frac{\frac{1}{\sqrt{Mm}}\Phi(A) + \sqrt{Mm}\,\Phi(A^{-1})}{2} \right)^r$$

$$\geqslant \frac{\frac{1}{(Mm)^{\frac{r}{2}}}\Phi(A)^r + (Mm)^{\frac{r}{2}}\Phi(A^{-1})^r}{2}$$

$$\geqslant \Phi(A)^r \sharp \Phi(A^{-1})^r,$$

where $0 \leqslant r \leqslant 1$, which nicely extend the operator Kantorovich inequality.

Mathematics subject classification (2010): Primary 47A63, Secondary 47A64, 15A60.

Keywords and phrases: Operator concave, operator inequalities, positive linear map, Kantorovich inequality, Bellman inequality.

REFERENCES

[1] T. ANDO, *Concavity of certain maps on positive definite matrices and applications to Hadamard products*, Linear Algebra Appl. **26** (1979), 203–241.

[2] T. ANDO AND F. HIAI, *Operator log-convex functions and operator means*, Math. Ann. **350**, 3 (2011), 611–630.

[3] R. BELLMAN, *On an inequality concerning an indefinite form*, Amer. Math. Monthly. **63** (1956), 108–109.

[4] R. BHATIA, *Positive definite matrices*, Princeton (NJ): Princeton University Press; 2007.

[5] E. Y. LEE, *A matrix reverse Cauchy-Schwarz inequality*, Linear Algebra Appl. **430**, 2 (2009), 805–810.

[6] M. LIN, *On an operator Kantorovich inequality for positive linear maps*, J. Math. Anal. Appl. **402**, 1 (2013), 127–132.

[7] A. W. MARSHALL AND I. OLKIN, *Matrix versions of Cauchy and Kantorovich inequalities*, Aequationes Math. **40** (1990), 89–93.

[8] J. MIĆIĆ, J. PEČARIĆ, Y. SEO AND M. TOMINAGA, *Inequalities for positive linear maps on Hermitian matrices*, Math. Inequal. Appl. **3**, 4 (2000), 559–591.

[9] H. R. MORADI, M. E. OMIDVAR, I. H. GÜMÜŞ AND R. NASERI, *A note on some inequalities for positive linear maps*, Linear Multilinear Algebra. **66**, 7 (2018), 1449–1460.

[10] A. MORASSAEI, F. MIRZAPOUR AND M. S. MOSLEHIAN, *Bellman inequality for Hilbert space operators*, Linear Algebra Appl. **438**, 10 (2013), 3776–3780.

[11] M. S. MOSLEHIAN, R. NAKAMOTO AND Y. SEO, *A Diaz-Metcalf type inequality for positive linear maps and its applications*, Electron. J. Linear Algebra. **22** (2011), 179–190.

[12] R. NAKAMOTO AND M. NAKAMURA, *Operator mean and Kantorovich inequality*, Math. Japon. **44**, 3 (1996), 495–498.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com