BOUNDEDNESS AND CONTINUITY OF MAXIMAL OPERATORS ASSOCIATED TO POLYNOMIAL COMPOUND CURVES ON TRIEBEL–LIZORKIN SPACES

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Abstract. In this paper we study the Triebel-Lizorkin space boundedness and continuity of maximal operators related to rough singular integrals associated to polynomial compound curves. We prove that the above operators are bounded and continuous on the inhomogeneous Triebel-Lizorkin space $F_{p,q}^{\alpha}(\mathbb{R}^n)$ for all $\alpha \in (0,1)$, $p \in [2,\infty)$ and $q \in (2p/(p+2),\infty)$ under the conditions that the integral kernels are given by $\Omega \in L(\log^+ L)^{1/2}(\mathrm{S}^{n-1}) \cup (\cup_{1<r<\infty} B_r^{0,-1/2}(\mathrm{S}^{n-1}))$. We also establish the boundedness and continuity of the above operators on the inhomogeneous Besov space $B_{p,q}^{\alpha}(\mathbb{R}^n)$ for $\alpha \in (0,1)$, $p \in [2,\infty)$ and $q \in (1,\infty)$. In addition, the corresponding results for maximal operators related to parametric Marcinkiewicz integrals are also considered.

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