A NEW VARIABLE EXPONENT PICONE IDENTITY AND APPLICTIONS

TINGFU FENG AND JUNQIANG HAN

Abstract. In this paper, we derive a new variable exponent Picone identity for $p(x)$-Laplacian, which contains some known Picone identities. As applications, a strict monotonicity of principal eigenvalues with respect to domains for the eigenvalue problems to $p(x)$-Laplace equation, a variable exponent Barta type inequality, a variable exponent Hardy type inequality with weight, a Sturmian comparison principle to $p(x)$-Laplace equation and a Liouville type theorem to $p(x)$-Laplace system are shown.

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