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WINTGEN INEQUALITY FOR STATISTICAL SURFACES

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Abstract. The Wintgen inequality (1979) is a sharp geometric inequality for surfaces in the 4-dimensional Euclidean space involving the Gauss curvature (intrinsic invariant) and the normal curvature and squared mean curvature (extrinsic invariants), respectively. In the present paper we obtain a Wintgen inequality for statistical surfaces.

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