MONOTONICITY OF THE JENSEN FUNCTIONAL FOR $f-$ DIVERGENCES WITH APPLICATIONS TO THE ZIPF–MANDELBROT LAW

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Abstract. The Jensen functional in its discrete form is brought in relation to the Csiszár divergence functional via its monotonicity property. Thus deduced general results branch into specific forms for some of the well known $f-$ divergences, e.g. the Kullback-Leibler divergence, the Hellinger distance, the Bhattacharyya coefficient, χ^2- divergence, total variation distance. Obtained comparative inequalities are also interpreted in the environment of the Zipf and the Zipf-Mandelbrot law.

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