

## ESTIMATES ON THE GAP IN BULLEN'S INEQUALITY

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*Abstract.* Some companions and estimates of the gap in Bullen's inequality are shown. The results are applied to special means and connections with the methods of computational fluid dynamics are indicated.

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