

ON ITERATED AND BILINEAR INTEGRAL HARDY–TYPE OPERATORS

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Abstract. We characterize the weighted inequalities on Lebesgue cone of all nonnegative functions on the semi-axis for iterated integral operators with Oinarov kernels.

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