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# FULL RANGE BOUNDEDNESS OF BILINEAR HILBERT TRANSFORM ALONG CERTAIN POLYNOMIALS

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*Abstract.* Let $P$ and $Q$ be two polynomials without constant term. Assume that the operator $B_{P,Q}(f,g)(x)=\int f(x-P(t))g(x-Q(t))\frac{dt}{t}$ is bounded from $L^{p_1}\times L^{p_2}$ into $L^r$, $p_1,p_2\in(1,\infty)$, $\frac{1}{p_1}+\frac{1}{p_2}=\frac{1}{r}$. It is proved that if $P'(t)>0$ for all $t\neq 0$, then $r\geqslant\frac{d}{d+1}$. Here $d$ is the correlation degree of $P$ and $Q$ which is defined as the largest multiplicity of non-zero real roots of $P'-Q'$.

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