

A HARDY-TYPE INEQUALITY WITH AHARONOV-BOHM MAGNETIC FIELD ON THE POINCARÉ DISK

LI ZHU

Abstract. A version of Aharonov-Bohm magnetic field on the Poincaré disk is introduced, then a Hardy-type inequality with Aharonov-Bohm magnetic field is proved.

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