

ON BENNETT'S CONJECTURE AND COMPLETE MONOTONICITY

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Abstract. Bennett [1] gave a generalization of Schur's theorem in order to study various moment-preserving transformations. Recently, Su [5] confirmed a monotonicity conjecture of Bennett which is related to the generalized Schur's theorem and Haber's inequality. In this paper we present a short proof of this result which is based on a combinatorial identity. Moreover, we show that the function in Bennett's conjecture is not only monotonically decreasing but completely monotonic. Furthermore, we give its explicit representation as a Laplace integral which implies the complete monotonicity. Finally, we prove a multivariate version of the above-mentioned combinatorial identity.

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