EXTENSIONS OF FEFFERMAN–STEIN MAXIMAL INEQUALITIES

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Abstract. Let $\beta_1, \ldots, \beta_m \in [0, \infty)$ and $\mathscr{M}_{L(\log L)^{\vec{\beta}}}$ be the maximal operator defined by

$$\mathscr{M}_{L(\log L)^{\vec{\beta}}}(f_1, \ldots, f_m)(x) = \sup_{Q \ni x} \prod_{j=1}^{m} \|f_j\|_{L(\log L)^{\beta_j}, Q}.$$

In this paper, we establish the weighted bounds in terms of the $A_{\vec{P}}(\mathbb{R}^{mn})$ constant for $\mathscr{M}_{L(\log L)^{\vec{\beta}}}$ from $L^{p_1}(l^{q_1}; \mathbb{R}^n, w_1) \times \ldots \times L^{p_m}(l^{q_m}; \mathbb{R}^n, w_m)$ to $L^p(l^q; \mathbb{R}^n, v_{\vec{w}})$, where $p_1, \ldots, p_m, q_1, \ldots, q_m \in (1, \infty)$, $1/p = 1/p_1 + \ldots + 1/p_m$, $1/q = 1/q_1 + \ldots + 1/q_m$ and $\vec{w} = (w_1, \ldots, w_m)$ a multiple $A_{\vec{P}}$ weights. A weak type endpoint inequality for vector-valued operator $\mathscr{M}_{L(\log L)^{\vec{\beta}}}$ is also given.

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