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ON THEOREMS OF MORGAN AND COWLING–PRICE FOR SELECTED NILPOTENT LIE GROUPS

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Abstract. Let G be a connected, simply connected nilpotent Lie group. For $p,q \in [1,+\infty]$, the $L^p - L^q$ analogue of Morgan's theorem was proved only for two step nilpotent Lie groups. In order to study this problem in larger subclasses, we formulate and prove a version of $L^p - L^q$ Morgan's theorem on nilpotent Lie groups whose Lie algebra admits an ideal which is a polarization for a dense subset of generic linear forms on the Lie algebra. A proof of an analogue of Cowling-Price Theorem is also provided in the same context.

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