

ON GENERALIZED CESÀRO STABLE FUNCTIONS

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Abstract. The notion of Cesàro stable function is generalized by introducing Cesàro mean of type $(b-1; c)$ which give rise to a new concept of generalized Cesàro stable function. As an application of generalized Cesàro stable functions we also prove for a convex function of order $\lambda \in [1/2, 1)$, its Cesàro mean of type $(b-1; c)$ is close-to-convex of order λ . Further two conjectures are also posed in the direction of generalized Cesàro stable function. Some particular cases of these conjectures are also discussed.

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