HEINZ MEANS AND TRIANGLES INSCRIBED
IN A SEMICIRCLE IN BANACH SPACES

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Abstract. In this paper, we introduce two classes of new geometric constants for Banach spaces by using the Heinz means that interpolate between the geometric and arithmetic means. One of these constants is closely related to the modulus of convexity of the space and it seems to represent a useful tool to estimate the exact values of the James and von Neumann-Jordan constants of some Banach spaces, while the study of the other one seems to be more complicated. Moreover, we investigate some geometric properties related to these constants and calculate the precise values of these two constants for several Banach spaces. We also study the stability under norm perturbations of these constants.

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