

## WEIGHTED MORREY ESTIMATES FOR HAUSDORFF OPERATOR AND ITS COMMUTATOR ON THE HEISENBERG GROUP

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*Abstract.* In this paper, we study the high-dimensional Hausdorff operators, defined via a general linear mapping  $A$ , and their commutators on the weighted Morrey spaces in the setting of the Heisenberg group. Particularly, under some assumption on the mapping  $A$ , we establish their sharp boundedness on the power weighted Morrey spaces.

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