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COMMUTATORS OF RIESZ POTENTIAL IN THE VANISHING GENERALIZED WEIGHTED MORREY SPACES WITH VARIABLE EXPONENT

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Abstract. Let $\Omega \subset \mathbb{R}^n$ be an unbounded open set. We consider the generalized weighted Morrey spaces $\mathcal{M}_{\omega}^{p(\cdot),\varphi}(\Omega)$ and the vanishing generalized weighted Morrey spaces $V\mathcal{M}_{\omega}^{p(\cdot),\varphi}(\Omega)$ with variable exponent $p(x)$ and a general function $\varphi(x,r)$ defining the Morrey-type norm. The main result of this paper are the boundedness of Riesz potential and its commutators on the spaces $\mathcal{M}_{\omega}^{p(\cdot),\varphi}(\Omega)$ and $V\mathcal{M}_{\omega}^{p(\cdot),\varphi}(\Omega)$. This result generalizes several existing results for Riesz potential and its commutators on Morrey type spaces. Especially, it gives a unified result for generalized Morrey spaces and variable Morrey spaces which currently gained a lot of attentions from researchers in theory of function spaces.

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