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ON THE MEASURE OF POLYNOMIALS ATTAINING MAXIMA ON A VERTEX

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Abstract. We calculate the probability that a k-homogeneous polynomial in n variables attain a local maximum on a vertex in terms of the "sharpness" of the vertex, and then study the dependence of this measure on the growth of dimension and degree. We find that the behavior of vertices with orthogonal edges is markedly different to that of sharper vertices. If the degree k grows with the dimension n, the probability that a polynomial attain a local maximum tends to $1/2$, but for orthogonal edges the growth-rate of k must be larger than $n\ln n$, while for sharper vertices a growth-rate larger than $\ln n$ will suffice.

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