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LANDAU–KOLMOGOROV TYPE INEQUALITIES FOR CURVES ON RIEMANNIAN MANIFOLDS

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Abstract. We obtain Landau-Kolmogorov type inequalities for mappings defined on the whole real axis and taking values in Riemannian manifolds. In terms of an auxiliary convex function, we find conditions under which the boundedness of covariant derivative along the curve under consideration ensures the boundedness of the corresponding tangent vector field. We use the square of the distance function as the auxiliary one to establish counterparts of the Landau – Hadamard and the Landau-Kolmogorov inequalities where the norms of higher order derivatives of mapping are replaced, respectively, by the Chebyshev radius of curve and the corresponding iterates of covariant derivative along the curve.

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