

OPTIMAL ESTIMATES FOR THE FRACTIONAL HARDY OPERATOR ON VARIABLE EXPONENT LEBESGUE SPACES

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Abstract. Let $A_\alpha f(x) = \frac{1}{|B(0,|x|)|^{\alpha/n}} \int_{B(0,|x|)} f(t) dt$ be the n -dimensional fractional Hardy operator, where $0 < \alpha \leq n$. We prove optimality results for the action of the operator A_α on variable exponent Lebesgue spaces $L^{p(\cdot)}$ and weighted variable exponent Lebesgue spaces, as an extension of [13, 14, 17].

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