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DILATION–COMMUTING OPERATORS ON POWER–WEIGHTED ORLICZ CLASSES

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Abstract. Let Φ be a nondecreasing function from $\mathbb{R}_+ = (0,\infty)$ onto itself. Fix $\gamma \in \mathbb{R} = (-\infty,\infty)$ and let $L_{\Phi,t^\gamma}(\mathbb{R}_+)$ be the set of all Lebesgue-measurable functions f from $\mathbb{R}_+$ to $\mathbb{R}$ for which

$$\int_{\mathbb{R}_+} \Phi(k|f(t)|)\, t^\gamma dt < \infty$$

for some $k > 0$. Define the gauge ρ_{Φ,t^γ} at $f \in L_{\Phi,t^\gamma}(\mathbb{R}_+)$ by

$$\rho_{\Phi,t^\gamma}(f) = \inf\left\{\lambda > 0 : \int_{\mathbb{R}_+} \Phi\left(\frac{|f(t)|}{\lambda}\right)\frac{t^\gamma}{\lambda} dt \leqslant 1\right\}.$$

Our principal goal in this paper is to find conditions on the nondecreasing functions Φ_1 and Φ_2, $\gamma \in \mathbb{R}$ and an operator T so that the assertions

$$\rho_{\Phi_1,t^\gamma}(Tf) \leqslant C\rho_{\Phi_2,t^\gamma}(f) \tag{G}$$

and

$$\int_{\mathbb{R}_+} \Phi_1(|(Tf)(t)|)\, t^\gamma dt \leqslant K \int_{\mathbb{R}_+} \Phi_2(K|f(s)|)\, s^\gamma ds, \tag{M}$$

concerning $f \in S(\mathbb{R}_+)$, the class of simple functions supported in $\mathbb{R}_+$, are equivalent and to then find necessary and sufficient conditions in order that (M) holds.

In addition, we investigate the connection between (G) and the assertion that

$$T : \mathring{L}_{\Phi_2,t^\gamma}(\mathbb{R}_+) \to L_{\Phi_1,t^\gamma}(\mathbb{R}_+),$$

where $\mathring{L}_{\Phi_2,t^\gamma}(\mathbb{R}_+)$ is the closure of $S(\mathbb{R}_+)$ in $L_{\Phi_2,t^\gamma}(\mathbb{R}_+)$.

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REFERENCES

[1] S. BLOOM AND R. KERMAN, *Weighted L_Φ integral inequalities for operators of Hardy type*, Studia Math. **110**, 1 (1994), 35–52.
[2] S. BLOOM AND R. KERMAN, *Weighted Orlicz space integral inequalities for the Hardy-Littlewood maximal operator*, Studia Math. **110**, 2 (1994), 149–167.
[3] C. BENNETT AND R. SHARPLEY, *Interpolation of operators*, Pure and Applied Mathematics, vol. **129**, Academic Press Inc., Boston, MA, 1988.
[4] A. CIANCHI, *Hardy inequalities in Orlicz spaces*, Trans. Amer. Math. Soc. **351**, 6 (1999), 2459–2478.
[5] R. COIFMAN AND C. FEFFERMAN, *Weighted norm inequalities for maximal functions and singular integrals*, Studia Math. **51**, 3 (1974), 241–250.
[6] M. FABIAN, P. HABALA, P. HÁJEK, V. MONTESINOS AND V. ZIZLER, *Banach space theory*, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, Springer, 2011.
[7] R. KERMAN AND L. PICK, *Explicit formulas for optimal rearrangement-invariant norms in Sobolev imbedding inequalities*, Studia Math. **206**, 2 (2011), 97–119.

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[8] R. KERMAN AND A. TORCHINSKY, *Integral inequalities with weights for the Hardy maximal function*, Studia Math. **71**, 3 (1982), 277–284.

[9] L. MALIGRANDA, *Orlicz spaces and interpolation*, Sem. Math. 5, Dep. Mat., Univ. Estadual de Campinas, Campinas SP, Brazil (1989).

[10] W. MATUSZEWSKA AND W. ORLICZ, *On certain properties of φ-functions*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **8** (1960), 439–443.

[11] S. MAZUR AND W. ORLICZ, *On some classes of linear spaces*, Studia Math. **17** (1958), 97–119.

[12] W. ORLICZ, *A note on modular spaces. I*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **9** (1961), 157–162.

[13] A. TORCHINSKY, *Real-variable methods in harmonic analysis*, Pure and Applied Mathematics, vol. **123**, Academic Press, Inc., Orlando, FL, 1986.

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