WEIGHTED NORM INEQUALITIES FOR
PARAMETRIC LITTLEWOOD–PALEY OPERATORS

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Abstract. In this paper, we establish the boundedness of parametric Littlewood-Paley operators from Musielak-Orlicz Hardy space to Musielak-Orlicz space. The endpoint weak type estimates are also obtained. Part of these results are new even for classical Hardy space of Fefferman and Stein.

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