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INTERVAL–TYPE THEOREMS CONCERNING QUASI–ARITHMETIC MEANS

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Abstract. Family of quasi-arithmetic means has a natural, partial order (point-wise order) $A^{[f]} \leqslant A^{[g]}$ if and only if $A^{[f]}(v) \leqslant A^{[g]}(v)$ for all admissible vectors v (f, g and, later, h are continuous, monotone and defined on a common interval).

Therefore one can introduce the notion of interval-type sets (sets $\mathscr{I}$ such that whenever $A^{[f]} \leqslant A^{[h]} \leqslant A^{[g]}$ for some $A^{[f]}, A^{[g]} \in \mathscr{I}$ then $A^{[h]} \in \mathscr{I}$ too).

Our aim is to give examples of interval-type sets involving vary smoothness assumptions of generating functions.

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