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A NEW COUNTEREXAMPLE TO SANGWINE–YAGER'S CONJECTURE

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Abstract. Sangwine-Yager conjectured in [9] that if $r_1 \leqslant \ldots \leqslant r_n$ are the real parts of the roots of the (formal) alternating Steiner polynomial of $V(K-tE)$, then $0 < r_1 \leqslant r(K;E) \leqslant R(K;E) \leqslant r_n$, where $r(K;E)$ and $R(K;E)$ are the inradius, respectively, outradius, or circumradius, of K relative to E. We present here a new counterexample to this conjecture in dimension 3 when none of the bodies is a Euclidean ball. Previous examples due to Henk and Hernández Cifre, and, respectively, to Hernández Cifre and Saorín, were constructed with fairly technical tools. Our example is non-trivial in the sense that both K and E are top dimensional convex bodies, yet it is easy to present.

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