CHARACTERIZATION OF OPERATOR CONVEX FUNCTIONS BY CERTAIN OPERATOR INEQUALITIES

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Abstract. For \( \lambda \in (0,1) \), let \( \psi \) be a non-constant, non-negative, continuous function on \((0,\infty)\) and let \( \Gamma_\lambda(\psi) \) be the set of all non-trivial operator means \( \sigma \) such that an inequality
\[
\psi(A \nabla_\lambda B) \leq \psi(A) \sigma \psi(B)
\]
holds for all \( A, B \in B(H)^{++} \). Then we have:

1. \( \psi \) is a decreasing operator convex function if and only if
   \[
   \Gamma_\lambda(\psi) = \{ \sigma \mid !_\lambda \leq \sigma \leq \nabla_\lambda \}.
   \]

2. \( \psi \) is an operator convex function which is not a decreasing function if and only if
   \[
   \Gamma_\lambda(\psi) = \{ \nabla_\lambda \}.
   \]

The first result is a weighted version of Ando and Hiai’s characterization of an operator monotone decreasing function and these two results imply each other.


Keywords and phrases: Operator means, operator monotone functions, operator convex functions.

REFERENCES