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BOUNDEDNESS OF GENERALIZED RIESZ POTENTIALS OF FUNCTIONS IN MORREY SPACES $L^{(1,\varphi;\kappa)}(G)$ OVER NON–DOUBLING MEASURE SPACES

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Abstract. Our aim in this paper is to deal with the boundedness of generalized Riesz potentials $I_{\rho,\mu,\tau}f$ of functions in Morrey spaces $L^{(1,\varphi;\kappa)}(G)$ over non-doubling measure spaces, as an extension of [4, 6, 9, 12, 19]. The local integrability is assumed to be minimal, so that the results can not be obtained by the Hardy-Littlewood maximal operator. What is new in this paper is that φ depends on $x \in X$ and that the underlying measure μ is not doubling.

Mathematics subject classification (2010): 26A33, 31B15, 46E30, 46E35.
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Paper MIA-22-41

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