

## COMMUTATORS OF FRACTIONAL INTEGRALS ON MARTINGALE MORREY SPACES

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**Abstract.** On martingale Morrey spaces we give necessary and sufficient conditions for the boundedness and compactness of the commutator generated by the fractional integral and a function in the martingale Campanato space. We also give the conditions for the boundedness and compactness from martingale Morrey spaces to martingale Triebel-Lizorkin-Morrey spaces.

**Mathematics subject classification (2010):** Primary 60G46, Secondary 42B30, 46E30, 42B35.

**Keywords and phrases:** Martingale, Morrey space, fractional integral, commutator.

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