

NEW CONVOLUTIONS WEIGHTED BY HERMITE FUNCTIONS AND THEIR APPLICATIONS

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Abstract. We introduce eight new convolutions weighted by multi-dimensional Hermite functions, prove two Young-type inequalities, and exhibit their applications in different subjects. One application consists in the study of the solvability of a very general class of integral equations whose kernel depends on four different functions. Necessary and sufficient conditions for the unique solvability of such integral equations are here obtained.
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