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ZERO–ORDER MEHLER–FOCK TRANSFORM AND SOBOLEV–TYPE SPACE

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Abstract. The present paper is devoted to the study of the Mehler-Fock transform with index as the Legendre function of first kind. Continuity property of the Mehler-fock transform on the test function spaces Λ_α and $\mathscr{G}_\alpha$ is given. Moreover pseudo-differential operator (p.d.o.) with symbol $\sigma(x,\tau) \in S^m$ in terms of Mehler-Fock transform is defined and also its continuity property from test function space $\mathscr{G}_\alpha$ into Λ_α is shown. The Mehler-Fock potential (MF-potential) $\mathscr{P}^s_\sigma$ is defined on $\mathscr{G}_\alpha(I)$ space and it is extended to the space of distribution. Also some properties of MF-potential are discussed. At the end Sobolev type space $V^{s,p}(I)$ is defined and it is shown that MF-potential is an isometry of $V^{s,p}(I)$.

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