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REMARKS ON TWO DETERMINANTAL INEQUALITIES

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Abstract. Denote by $\mathbb{P}_n$ the set of $n \times n$ positive definite matrices. Let $D = D_1 \oplus \ldots \oplus D_k$, where $D_1 \in \mathbb{P}_{n_1}, \ldots, D_k \in \mathbb{P}_{n_k}$ with $n_1 + \cdots + n_k = n$. Partition $C \in \mathbb{P}_n$ according to $(n_1, \ldots, n_k)$ so that $\mathrm{Diag}\, C = C_1 \oplus \ldots \oplus C_k$. For any $p \geqslant 0$, we have

$$\det(I_{n_1} + (C_1^{-1}D_1)^p) \cdots \det(I_{n_k} + (C_k^{-1}D_k)^p) \leqslant \det(I_n + (C^{-1}D)^p).$$

This is a generalization of a determinantal inequality of Matic [6, Theorem 1.1]. In addition, we obtain a weak majorization result which is complementary to a determinantal inequality of Choi [2, Theorem 2] and ask a weak log majorization open question.

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