

APPROXIMATION BY MARCINKIEWICZ Θ -MEANS OF DOUBLE WALSH-FOURIER SERIES

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Abstract. In this article we discuss the behaviour of Θ -means of quadratical partial sums of double Walsh series of a function in $L^p(G^2)$ ($1 \leq p < \infty$). In case $p = \infty$ by $L^p(G^2)$ we mean C , the collection of continuous functions on G^2 . We present the rate of the approximation by Θ -means, in particular, in $\text{Lip}(\alpha, p)$, where $\alpha > 0$ and $1 \leq p < \infty$.

Our main theorem generalizes two result of Nagy on Nörlund means and weighted means of the cubical partial sums of double Walsh-Fourier series [15, 16]. Specifically, we give the two-dimensional analogue of the two results of Móricz, Siddiqi on Nörlund means [14] and Móricz, Rhoades on weighted means [12].

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REFERENCES

- [1] G.H. AGAEV, N.JA. VILENKIN, G.M. DZHAFARLI, AND A.I. RUBINSTEIN, *Multiplicative systems of functions and harmonic analysis on 0-dimensional groups*, Izd. ("ELM"), Baku, (1981) (in Russian).
- [2] L. BARAMIDZE, L.-E. PERSSON, G. TEPHNADZE AND P. WALL, *Sharp $H_p - L_p$ type inequalities of weighted maximal operators of Vilenkin-Nörlund means and its applications*, Journal of Inequalities and Applications, **242**, (2016).
- [3] I. BLAHOTA AND K. NAGY, *Approximation by Θ -means of Walsh-Fourier series*, Analysis Mathematica, **44**, (1) (2018), 57–71.
- [4] I. BLAHOTA AND G. TEPHNADZE, *On the Nörlund means of Vilenkin-Fourier series*, Acta Mathematica Academiae Paedagogicae Nyíregyháziensis, **32**, (2) (2016), 203–213.
- [5] Á. CHRIPKÓ, *Weighted approximation via Θ -summations of Fourier-Jacobi series*, Studia Sci. Math. Hungar., **47**, (2) (2010), 139–154.
- [6] T. EISNER, *The Θ -summation on local fields*, Annales Universitatis Scientiarum Budapestinensis de Rolando Eotvos Nominatae Sectio Computatorica, **33**, (2011), 137–160.
- [7] S. FRIDL, P. MANCHANDA, AND A.H. SIDDIQI, *Approximation by Walsh-Nörlund means*, Acta Sci. Math., **74**, (2008), 593–608.
- [8] V.A. GLUKHOV, *On the summability of multiple Fourier series with respect to multiplicative systems*, Mat. Zametki, **39**, (1986), 665–673 (in Russian).
- [9] U. GOGINA, *On the approximation properties of Cesàro means of negative order of Walsh-Fourier series*, J. Approx. Theory, **115**, (2002), 9–20.
- [10] M.A. JASTREBOVA, *On approximation of functions satisfying the Lipschitz condition by arithmetic means of their Walsh-Fourier series*, Mat. Sb., **71**, (1966), 214–226 (in Russian).
- [11] N. MEMIĆ, L.-E. PERSSON AND G. TEPHNADZE, *A note on the maximal operators of Vilenkin-Nörlund means with non-increasing coefficients*, Studia Scientiarum Mathematicarum Hungarica, **53**, (4) (2016), 545–556.
- [12] F. MÓRICZ AND B. E. RHOADES, *Approximation by weighted means of Walsh-Fourier series*, Int. J. Math. Sci., **19**, (1) (1996), 1–8.

- [13] F. MÓRICZ AND F. SCHIPP, *On the integrability and L^1 -convergence of Walsh series with coefficients of bounded variation*, J. Math. Anal. Appl., **146**, (1) (1990), 99–109.
- [14] F. MÓRICZ AND A. SIDDIQI, *Approximation by Nörlund means of Walsh-Fourier series*, J. Approx. Theory, **70**, (1992), 375–389.
- [15] K. NAGY, *Approximation by Nörlund means of Walsh-Kaczmarz-Fourier series*, Georgian Math. J., **18**, (1) (2011), 147–162.
- [16] K. NAGY, *Approximation by weighted means of Walsh-Kaczmarz-Fourier series*, Rendiconti del Circolo Matematico di Palermo, Serie II, **82**, (2010), 387–406.
- [17] K. NAGY, *Approximation by Nörlund means of double Walsh-Fourier series for Lipschitz functions*, Math. Ineq. Appl., **15**, (2) (2012), 301–322.
- [18] F. SCHIPP, W. R. WADE, P. SIMON, AND J. PÁL, *Walsh Series. An Introduction to Dyadic Harmonic Analysis*, Adam Hilger, (Bristol-New York 1990).
- [19] V.A. SKVORTSOV, *Certain estimates of approximation of functions by Cesàro means of Walsh-Fourier series*, Mat. Zametki, **29**, (1981), 539–547 (in Russian).
- [20] R. TOLEDO, *On the boundedness of the L^1 -norm of Walsh-Fejer kernels*, Journal of Math. Anal. and Appl., **457**, (1) (2018), 153–178.
- [21] F. WEISZ, *Θ -summability of Fourier series*, Acta Math. Hungar., **103**, (1-2) (2004), 139–175.
- [22] F. WEISZ, *Θ -summation and Hardy spaces*, J. Approx. Theory, **107**, (2000), 121–142.
- [23] F. WEISZ, *Several dimensional Θ -summability and Hardy spaces*, Math. Nachr., **230**, (2001), 159–180.
- [24] F. WEISZ, *Marcinkiewicz- Θ -summability of double Fourier series*, Annales Univ. Sci. Budapest., Sect. Comp., **24**, (2004), 103–118.
- [25] C. WATARI, *Best approximation by Walsh polynomials*, Tohoku Math. J., **15**, (1963), 1–5.
- [26] SH. YANO, *On Walsh-Fourier series*, Tohoku Math. J., **3**, (1951), 223–242.
- [27] SH. YANO, *On approximation by Walsh functions*, Proc. Amer. Math. Soc., **2**, (1951), 962–967.
- [28] A. ZYGMUND, *Trigonometric series*, 3rd edition, Vol. 1 & 2 and combined, Cambridge Univ. Press, (2015).