

THE REVERSED HARDY–LITTLEWOOD–SOBOLEV TYPE INTEGRAL SYSTEMS WITH WEIGHTS

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Abstract. This paper is concerned with the existence of positive entire solutions of a weighted integral system. Such a system comes from the conformal properties of the reversed Hardy–Littlewood–Sobolev inequality. Several sufficient conditions of the existence/nonexistence are presented.

Mathematics subject classification (2010): 26D15, 45E10, 45G05, 45M20.

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