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TWO TRACE INEQUALITIES FOR OPERATOR FUNCTIONS

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Abstract. In this paper we show that for a non-negative operator monotone function f on $[0,\infty)$ such that $f(0)=0$ and for any positive semidefinite matrices A and B,

$$\mathrm{Tr}\,((A-B)(f(A)-f(B)))\leqslant \mathrm{Tr}\,(|A-B|f(|A-B|)).$$

When the function f is operator convex on $[0,\infty)$, the inequality is reversed.

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