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INEQUALITIES FOR CERTAIN POWERS OF SEVERAL POSITIVE DEFINITE MATRICES

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Abstract. Let $A_i, i = 1,...,m,$ and X be $n \times n$ matrices such that each A_i is positive definite with $0 < a_i \leqslant s_n(A_i)$ and X is Hermitian. Then it is shown that

$$\left|\left|\left| \left(\sum_{i=1}^{m} A_i^{a_{m+1-i}} \right) X + X \left(\sum_{i=1}^{m} A_{m+1-i}^{a_i} \right) \right|\right|\right| \geqslant m(1+l^2) \, |||X|||,$$

for every unitarily invariant norm, where $l = \min\limits_{1 \leqslant i \leqslant m} a_i$.

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