Mathematical
Inequalities
& Applications
Volume 22, Number 4 (2019), 1123–1136
doi:10.7153/mia-2019-22-77

ON WEIGHTED QUASI–ARITHMETIC MEANS WHICH ARE CONVEX

JACEK CHUDZIAK, DOROTA GŁAZOWSKA, JUSTYNA JARCZYK AND WITOLD JARCZYK

Abstract. We study convexity in the class of weighted quasi-arithmetic means. It turns out that their convexity depends only on the generator, neither on weights, nor on the number of variables. Connections between the convexity of a mean and the convexity of its increasing generators are considered. We prove that convex means are generated by convex strictly increasing functions. A simple example shows that the converse is not true, so the problem arises when this is the case. Some answers are given under regularity assumptions imposed on the generator.

Mathematics subject classification (2010): 26E60, 26B25, 39B62.
Keywords and phrases: Weighted quasi-arithmetic mean, convexity.

REFERENCES

[1] F. BERNSTEIN, G. DOETSCH, *Zur Theorie der konvexen Funktionen*, Math. Ann. **76** (1915), 514–526.
[2] Z. DARÓCZY, ZS. PÁLES, *Convexity with given infinite weight sequences*, Stochastica **11** (1987), 5–12.
[3] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, Cambridge University Press, Cambridge, 1934 (1st edition), 1952 (2nd edition).
[4] M. KUCZMA, *An introduction to the theory of functional equations and inequalities. Cauchy's equation and Jensen's inequality* (2nd edition), edited and with preface by Attila Gilányi, Birkhaüser, Basel, 2009.
[5] N. KUHN, *A note on t -convex functions*, General Inequalities **4** (Oberwolfach, 1983), 269–276, Internat. Schriftenreihe Numer. Math. 71, Birkhaüser, Basel, 1984.
[6] L. LOSONCZI, *Subadditive mittelverte*, Arch. Math. (Basel) **22** (1971), 168–174.
[7] GY. MAKSA, ZS. PÁLES, *Remarks on the comparison of weighted quasi-arithmetic means*, Colloq. Math. **120** (2010), 77–84.
[8] T. MAŁOLEPSZY, oral communication.
[9] ZS. PÁLES, *General inequalities for quasideviation means*, Aequationes Math. **36** (1988), 32–56.
[10] ZS. PÁLES, P. PASTECZKA, *On the best Hardy constant for quasi-arithmetic means and homogeneous deviation means*, Math. Inequal. Appl. **21** (2018), 585–599.

Paper MIA-22-77