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A GENERALIZATION OF G–METRIC SPACES AND RELATED FIXED POINT THEOREMS

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Abstract. The idea of b-metric was proposed from the works of Bourbaki and Bakhtin. Czerwik gave an axiom which was weaker than the triangular inequality and formally defined b-metric spaces with a view of generalizing the Banach contraction mapping theorem. Further, in 2006, Mustafa and Sims have introduced an alternative more robust generalization of metric spaces to overcome fundamental flaws in B.C. Dhage$'s$ theory of generalized metric spaces and named it as G-metric spaces. In this paper, inspired by the concept of b-metric spaces and G-metric spaces, a new generalization of G-metric spaces (named as G_b-metric spaces) are introduced that recovers a large class of topological spaces including standard metric spaces, b-metric spaces, G-metric spaces *etc*. In such spaces, a new version of known fixed point theorems in b-metric spaces as well as in G-metric spaces have been proved. As an application of our result, we establish an existence and uniqueness result for system of linear equations in G_b-complete metric spaces.

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Paper MIA-22-79

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