SOME INEQUALITIES ON w^*UR MODULUS OF CONVEXITY AND GEOMETRIC PROPERTIES OF BANACH SPACES X AND X^*

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Abstract. Let X be a Banach space. In this paper, we study the properties of w^*UR modulus of convexity of X^* respect to x, $\delta_X^*(\varepsilon,x)$, where $0 \leqslant \varepsilon \leqslant 2$ and $x \in S(X)$, and the relationship between the values of w^*UR modulus and reflexivity, uniform non-squareness and normal structure respectively. Among other results, we proved that if $\delta_X^*(\varepsilon,x) > \frac{1}{2} - \frac{\varepsilon}{4}$ for all $x \in S(X)$, and any $0 < \varepsilon < 2$ then both X and X^* have uniform normal structure.

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