

NEW BOUNDS FOR SHANNON, RELATIVE AND MANDELBROT ENTROPIES VIA ABEL–GONTSCHAROFF INTERPOLATING POLYNOMIAL

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Abstract. The Jensen’s inequality has tremendous implications in many fields of modern analysis. It helps computing useful upper bounds for several entropic measures used in information theory. We use discrete and continuous cyclic refinements of Jensen’s inequality and extend them from convex to higher order convex function by using new Green functions and Abel-Gontscharoff interpolating polynomial. As an application of our work, we establish connection among new entropic bounds for Shanon, Relative and Mandelbrot entropies.

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