

QUASI-CONVEX FUNCTIONS OF HIGHER ORDER

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Abstract. We introduce and investigate the notions of n -quasi-convex as well as strongly n -quasi-convex functions with modulus $c > 0$. We give characterizations of these functions, which are counterparts of those given for quasi-convex and strongly n -convex functions. We introduce and investigate the notions of n -quasi-concave and n -quasi-affine functions, as well as strongly n -quasi-concave and strongly n -quasi-affine functions. We also give a generalization of higher order quasi-convex functions introduced by E. Popoviciu (1982).

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