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SOBOLEV'S THEOREM FOR DOUBLE PHASE FUNCTIONALS

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Abstract. Our aim in this paper is to establish generalizations of Sobolev's theorem for double phase functionals $\Phi(x,t) = t^p + \{b(x)t(\log(e+t))^\tau\}^q$, where $1 < p \leqslant q < \infty$, $\tau > 0$ and b is a nonnegative bounded function satisfying $|b(x) - b(y)| \leqslant C|x-y|^\theta (\log(e+|x-y|^{-1}))^{-\tau}$ for $0 \leqslant \theta < 1$.

Mathematics subject classification (2010): 46E30, 42B25, 46E35.

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