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STOCHASTIC COMPARISONS OF THE LARGEST CLAIM AMOUNTS FROM TWO SETS OF INTERDEPENDENT HETEROGENEOUS PORTFOLIOS

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Abstract. Let $X_{\lambda_1},\dots,X_{\lambda_n}$ be continuous and dependent non-negative random variables and $Y_i = I_{p_i}X_{\lambda_i}$, $i=1,\dots,n$, where $I_{p_1},\dots,I_{p_n}$ are independent Bernoulli random variables independent of X_{λ_i}'s, with $\mathrm{E}[I_{p_i}] = p_i$, $i=1,\dots,n$. In actuarial sciences, Y_i corresponds to the claim amount in a portfolio of risks. In this paper, we compare the largest claim amounts of two sets of interdependent portfolios, in the sense of usual stochastic order, when the variables in one set have the parameters $\lambda_1,\dots,\lambda_n$ and $p_1,\dots,p_n$ and the variables in the other set have the parameters $\lambda_1^*,\dots,\lambda_n^*$ and $p_1^*,\dots,p_n^*$. For illustration, we apply the results to some important models in actuary.

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