ON THE VARIATION OF THE DISCRETE MAXIMAL OPERATOR

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Abstract. In this note we study the endpoint regularity properties of the discrete nontangential fractional maximal operator

\[ M_{\alpha, \beta} f(n) = \sup_{r \in \mathbb{N}} \frac{1}{(2r+1)^{1-\alpha}} \sum_{k=-r}^{r} |f(m+k)|, \]

where \( \alpha \in [0, 1) \), \( \beta \in [0, \infty) \) and \( \mathbb{N} = \{0, 1, 2, \ldots\} \), covering the discrete centered Hardy-Littlewood maximal operator and its fractional variant. More precisely, we establish the sharp boundedness and continuity for \( M_{\alpha, \beta} \) from \( \ell^1(\mathbb{Z}) \) to \( BV(\mathbb{Z}) \). When \( \alpha = 0 \), we prove that the operator \( M_{\alpha, \beta} \) is bounded and continuous on \( BV(\mathbb{Z}) \). Here \( BV(\mathbb{Z}) \) denotes the set of functions of bounded variation defined on \( \mathbb{Z} \). Our main results represent generalizations as well as natural extensions of many previously known ones.


Keywords and phrases: Discrete nontangential fractional maximal operator, bounded variation, boundedness, continuity.

REFERENCES


