SOME INEQUALITIES AND AN APPLICATION
OF EXPONENTIAL POLYNOMIALS

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Abstract. In the paper, with the help of the Faà di Bruno formula, properties of the Bell polynomials of the second kind, and the inversion theorem for the Stirling numbers of the first and second kinds, the author presents an explicit formula and an identity for higher order derivatives of generating functions of exponential polynomials; consequently, the author recovers an explicit formula and finds an identity for exponential polynomials in terms of the Stirling numbers of the fist and second kinds; furthermore and importantly, with the assistance of the complete monotonicity of generating functions of exponential polynomials and other known conclusions, the author constructs some determinantal inequalities and product inequalities and deduces the logarithmic convexity and logarithmic concavity of two sequences related to exponential polynomials; finally, the author gives an application of exponential polynomials by confirming that exponential polynomials satisfy conditions for sequences required in white noise distribution theory.

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Keywords and phrases: Exponential polynomial, Bell number, Touchard polynomial, Bell polynomial of the second kind, higher order derivative, generating function, Faà di Bruno formula, inversion theorem, Stirling number of the first kind, Stirling number of the second kind, explicit formula, absolute monotonicity, complete monotonicity, determinantal inequality, product inequality, logarithmic convexity, logarithmic concavity, white noise distribution theory.

REFERENCES

[1] N. ASAI, I. KUBO, AND H.-H. KUO, *Bell numbers, log-concavity, and log-convexity*, Acta Appl. Math. **63** (2000), no. 1-3, 79–87; Available online at https://doi.org/10.1023/A:1010738827855.

[2] N. ASAI, I. KUBO, AND H.-H. KUO, *Characterization of test functions in CKS-space*, Mathematical Physics and Stochastic Analysis (Lisbon, 1998), 68–78, World Sci. Publ., River Edge, NJ, 2000.

[3] N. ASAI, I. KUBO, AND H.-H. KUO, *General characterization theorems and intrinsic topologies in white noise analysis*, Hiroshima Math. J. **31** (2001), no. 2, 299–330; Available online at http://projecteuclid.org/euclid.hmj/1151105703.

[4] N. ASAI, I. KUBO, AND H.-H. KUO, *Roles of log-concavity, log-convexity, and growth order in white noise analysis*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **4** (2001), no. 1, 59–84; Available online at https://doi.org/10.1142/S0219025701000492.

[5] E. T. BELL, *Exponential numbers*, Amer. Math. Monthly **41** (1934), no. 7, 411–419; Available online at http://dx.doi.org/10.2307/2300300.

[6] E. A. BENDER AND E. R. CANFIELD, *Log-concavity and related properties of the cycle index polynomials*, J. Combin. Theory Ser. A **74** (1996), no. 1, 57–70; Available online at https://doi.org/10.1006/jcta.1996.0037.

[7] K. N. BOYADZHIEV, *Exponential polynomials, Stirling numbers, and evaluation of some gamma integrals*, Abstr. Appl. Anal. 2009, Art. ID 168672, 18 pp; Available online at http://dx.doi.org/10.1155/2009/168672.

[8] W. G. COCHRAN, H.-H. KUO, AND A. SENGUPTA, *A new class of white noise generalized functions*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **1** (1998), no. 1, 43–67; Available online at `https://doi.org/10.1142/S0219025798000053`.

[9] L. COMTET, *Advanced Combinatorics: The Art of Finite and Infinite Expansions*, Revised and Enlarged Edition, D. Reidel Publishing Co., Dordrecht and Boston, 1974; Available online at `https://doi.org/10.1007/978-94-010-2196-8`.

[10] A. DIL AND V. KURT, *Investigating geometric and exponential polynomials with Euler-Seidel matrices*, J. Integer Seq. **14** (2011), no. 4, Article 11.4.6, 12 pp.

[11] B.-N. GUO AND F. QI, *An explicit formula for Bell numbers in terms of Stirling numbers and hypergeometric functions*, Glob. J. Math. Anal. **2** (2014), no. 4, 243–248; Available online at `https://doi.org/10.14419/gjma.v2i4.3310`.

[12] I. KUBO, H.-H. KUO, AND A. SENGUPTA, *White noise analysis on a new space of Hida distributions*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **2** (1999), no. 3, 315–335; Available online at `https://doi.org/10.1142/S0219025799000199`.

[13] H.-H. KUO, *White Noise Distribution Theory*, Probability and Stochastics Series, CRC Press, Boca Raton, FL, 1996.

[14] W.-X. MA, *Bilinear equations, Bell polynomials and linear superposition principle*, J. Phys. Conf. Ser. **411** (2013), no. 1, Aricle ID 012021, 11 pages; Available online at `https://doi.org/10.1088/1742-6596/411/1/012021`.

[15] W.-X. MA, *Bilinear equations and resonant solutions characterized by Bell polynomials*, Rep. Math. Phys. **72** (2013), no. 1, 41–56; Available online at `https://doi.org/10.1016/S0034-4877(14)60003-3`.

[16] W.-X. MA, *Trilinear equations, Bell polynomials, and resonant solutions*, Front. Math. China **8** (2013), no. 5, 1139–1156; Available online at `https://doi.org/10.1007/s11464-013-0319-5`.

[17] F. MIRZAEE, *Numerical solution of nonlinear Fredholm-Volterra integral equations via Bell polynomials*, Comput. Methods Differ. Equ. **5** (2017), no. 2, 88–102.

[18] D. S. MITRINOVIĆ, *Analytic Inequalities*, Springer-Verlag, 1970.

[19] D. S. MITRINOVIĆ AND J. E. PEČARIĆ, *On two-place completely monotonic functions*, Anzeiger Öster. Akad. Wiss. Math.-Natturwiss. Kl. **126** (1989), 85–88.

[20] D. S. MITRINOVIĆ, J. E. PEČARIĆ, AND A. M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, 1993; Available online at `https://doi.org/10.1007/978-94-017-1043-5`.

[21] J. E. PEČARIĆ, *Remarks on some inequalities of A. M. Fink*, J. Math. Anal. Appl. **104** (1984), no. 2, 428–431; Available online at `https://doi.org/10.1016/0022-247X(84)90006-4`.

[22] F. QI, *An explicit formula for the Bell numbers in terms of the Lah and Stirling numbers*, Mediterr. J. Math. **13** (2016), no. 5, 2795–2800; Available online at `https://doi.org/10.1007/s00009-015-0655-7`.

[23] F. QI, *Integral representations for multivariate logarithmic polynomials*, J. Comput. Appl. Math. **336** (2018), 54–62; Available online at `https://doi.org/10.1016/j.cam.2017.11.047`.

[24] F. QI, *On multivariate logarithmic polynomials and their properties*, Indag. Math. (N.S.) **29** (2018), no. 5, 1179–1192; Available online at `https://doi.org/10.1016/j.indag.2018.04.002`.

[25] F. QI, *Some inequalities for the Bell numbers*, Proc. Indian Acad. Sci. Math. Sci. **127** (2017), no. 4, 551–564; Available online at `https://doi.org/10.1007/s12044-017-0355-2`.

[26] F. QI, *Some properties of the Touchard polynomials*, ResearchGate Working Paper (2017), available online at `https://doi.org/10.13140/RG.2.2.30022.16967`.

[27] F. QI, D. LIM, AND B.-N. GUO, *Explicit formulas and identities for the Bell polynomials and a sequence of polynomials applied to differential equations*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **113** (2019), no. 1, 1–9; Available online at `https://doi.org/10.1007/s13398-017-0427-2`.

[28] F. QI, D.-W. NIU, AND B.-N. GUO, *Some identities for a sequence of unnamed polynomials connected with the Bell polynomials*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Math. RACSAM **113** (2019), no. 2, 557–567; Available online at `https://doi.org/10.1007/s13398-018-0494-z`.

[29] F. QI, D.-W. NIU, D. LIM, AND B.-N. GUO, *Some properties and an application of multivariate exponential polynomials*, Math. Methods Appl. Sci. **43** (2020), in press; available online at `https://doi.org/10.1002/mma.6095`.

[30] F. Q I, X.-T. S HI, AND F.-F. L IU, *Expansions of the exponential and the logarithm of power series and applications*, Arab. J. Math. (Springer) **6** (2017), no. 2, 95–108; Available online at `https://doi.org/10.1007/s40065-017-0166-4`.

[31] J. Q UAINTANCE AND H. W. G OULD, *Combinatorial Identities for Stirling Numbers*, The unpublished notes of H. W. Gould. With a foreword by George E. Andrews. World Scientific Publishing Co. Pte. Ltd., Singapore, 2016.

[32] G.-C. R OTA, P. D OUBILET, C. G REENE, D. K AHANER, A. O DLYZKO, AND R. S TANLEY, *Finite Operator Calculus*, Academic Press, New York, NY, USA, 1975.

[33] J. T OUCHARD, *Nombres exponentiels et nombres de Bernoulli*, Canad. J. Math. **8** (1956), 305–320; Available online at `http://dx.doi.org/10.4153/CJM-1956-034-1`. (French)

[34] H. VAN H AERINGEN, *Completely monotonic and related functions*, J. Math. Anal. Appl. **204** (1996), no. 2, 389–408; Available online at `https://doi.org/10.1006/jmaa.1996.0443`.

[35] H. VAN H AERINGEN, *Inequalities for real powers of completely monotonic functions*, J. Math. Anal. Appl. **210** (1997), no. 1, 102–113; Available online at `https://doi.org/10.1006/jmaa.1997.5376`.

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