COMMUTATORS OF CERTAIN FRACTIONAL TYPE OPERATORS WITH HÖRMANDER CONDITIONS, ONE–WEIGHTED AND TWO–WEIGHTED INEQUALITIES

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Abstract. In this paper we study commutators of a certain class of fractional type integral operators. These operators are given by kernels of the form

$$K(x,y) = k_1(x - A_1 y)k_2(x - A_2 y)\dots k_m(x - A_m y),$$

where A_i are invertible matrices and each k_i satisfies a fractional size condition and generalized fractional Hörmander condition. We obtain weighted Coifman estimates and weighted $L^p(w^p)$ - $L^q(w^q)$ estimates. We also give a two-weighted strong type estimate for pairs of weights of the form (u, Su) where u is an arbitrary non-negative function and S is a maximal operator depending on the smoothness of the kernel K. For the singular case we also give a two-weighted endpoint estimate.

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