

HERMITE—HADAMARD TYPE INEQUALITY FOR CERTAIN SCHUR CONVEX FUNCTIONS

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Abstract. The main goal of this paper is to prove a Hermite-Hadamard type inequality for certain Schur convex functions using, as one of the main tools in the proof, a Korovkin-type approximation theorem.

Mathematics subject classification (2010): 26B25, 41A36, 26D15, 39B62.

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