

LINEAR MAPS OF POSITIVE PARTIAL TRANSPOSE MATRICES AND SINGULAR VALUE INEQUALITIES

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Abstract. Linear maps $\Phi : \mathbb{M}_n \rightarrow \mathbb{M}_k$ are called m -PPT if $[\Phi(A_{ij})]_{i,j=1}^m$ are positive partial transpose matrices for all positive semi-definite matrices $[A_{ij}]_{i,j=1}^m \in \mathbb{M}_m(\mathbb{M}_n)$. In this paper, connections between m -PPT maps, m -positive maps and m -copositive maps are given. In consequence, characterizations of completely PPT maps are obtained. The results are applied to study two linear maps $X \mapsto X + a(\text{tr}X)I$ and $X \mapsto a(\text{tr}X)I - X$ for $a \geq 0$. Moreover, singular values inequalities of 2×2 positive block matrices under these two linear maps are given. In particular, we prove an open singular values inequality formulated by Lin [Linear Algebra Appl, 520 (2017)] for $n \leq 3$.

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