LINEAR MAPS OF POSITIVE PARTIAL TRANSPOSE MATRICES AND SINGULAR VALUE INEQUALITIES

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Abstract. Linear maps $\Phi : M_n \rightarrow M_k$ are called $m$-PPT if $[\Phi(A_{ij})]_{i,j=1}^{m}$ are positive partial transpose matrices for all positive semi-definite matrices $[A_{ij}]_{i,j=1}^{m} \in M_m(M_n)$. In this paper, connections between $m$-PPT maps, $m$-positive maps and $m$-copositive maps are given. In consequence, characterizations of completely PPT maps are obtained. The results are applied to study two linear maps $X \mapsto X + a(\text{tr}X)I$ and $X \mapsto a(\text{tr}X)I - X$ for $a \geq 0$. Moreover, singular values inequalities of $2 \times 2$ positive block matrices under these two linear maps are given. In particular, we prove an open singular values inequality formulated by Lin [Linear Algebra Appl, 520 (2017)] for $n \leq 3$.


Keywords and phrases: Positive semi-definite matrices, positive partial transpose, singular values inequalities.

REFERENCES


